33-353 — Intermediate Optics Thursday, Oct-14, 2010

HW solution, week 8 due: Wednesday, Oct-20, 2010 – before class

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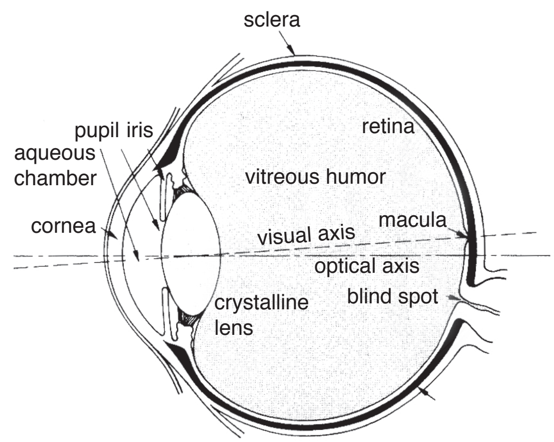
1. Human Eye **(4 pts)**

The human eye is quite complex, but in a rough approximation, a light ray travels from the outside air into a curved element, the cornea, and continues through the aqueous chamber, the eye lens and the vitreous chamber of the eyeball to the retina. Since all the tissues through which the ray travels have an index of $n_{tissue} \sim 1.37$ ($\pm$ 0.03), let us assume that the space posterior to the cornea is more or less homogeneous. The object focal length is f_o = 16 mm.



In the approximation specified above, determine

– the approximate curvature radius of the cornea,

– the location of the optical center of the imaging surface with respect to the cornea's vertex and

– the image focal length of the eye.

Same geometry as just one curved convex ($R > 0$) surface between two media ($n_1 \approx 1$, $n_1 = n_{tissue}$), thus

$$\frac{1}{s_o} + \frac{n_{tissue}}{s_1} = \frac{n_{tissue} - 1}{R} \text{ and}$$

$$f_o = R \cdot \frac{1}{n_{tissue} - 1} \; ; \; f_i = R \cdot \frac{n_{tissue}}{n_{tissue} - 1}$$

$$R = f_o \cdot (n_{tissue} - 1) \approx 5.6 \text{ mm}$$

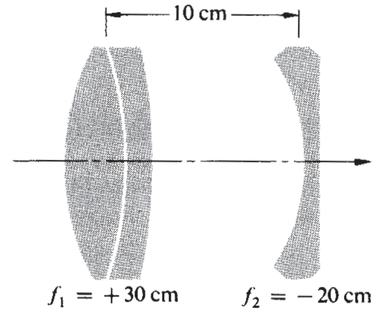
The optical center is the origin of the curvature radius, hence O is located 5.6 mm behind the vertex.

$$f_i = 5.6 \text{ mm} \cdot (1.37/0.37) \approx 21 \text{ mm}$$

(Because some of the optical elements have an index slightly larger than 1.37, the actual value of f_i is also somewhat larger, $f_i \approx 24$ mm.)

2. Lens Combination **(4 pts)**

Compute the image location and M_T for an object 30 cm from the front lens of the combination shown on the left, and sketch a ray diagram.

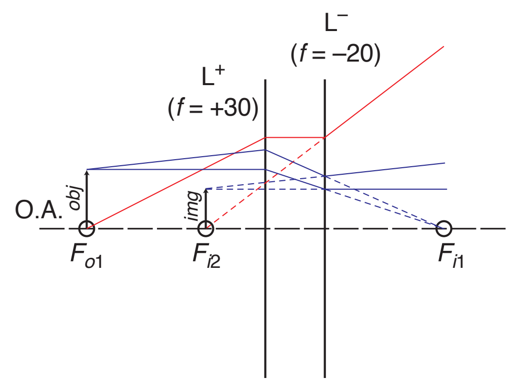


The object is in the focal plane of the doublet lens, lens 1. Therefore,

$\frac{1}{s_{i1}} = \frac{1}{f} - \frac{1}{s_{o1}} = 0$ yields $s_{i1} = \infty$: Any ray bundle emerging from the object is parallel after passing lens 1.

For lens 2 ($f_2 = -20$ cm), the lens equation $\frac{1}{s_{i2}} = \frac{1}{-20\text{ cm}} - \frac{1}{-\infty}$

yields $s_{i2} = -20$ cm, i.e., the image is virtual and erect, and located $d = -10$ cm to the left of lens 1, in the image focus of lens 2.



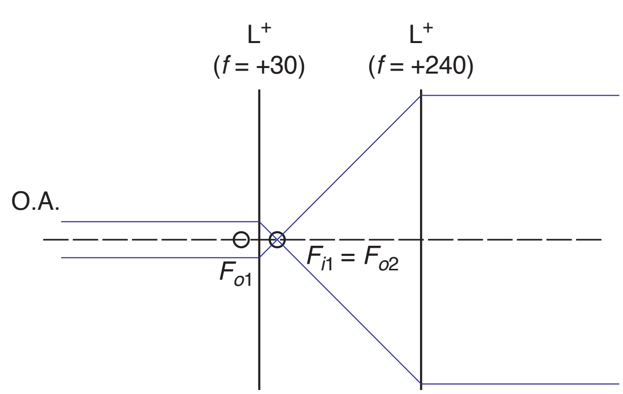
$$M_T = \frac{f_1 s_{i2}}{(s_{o1} - f_1)d - s_{o1} f_1} = \frac{30 \cdot -20}{(30-30)10 - 30} = 2/3$$

(all measures in cm)

3. Laser Beam Expander **(3 pts)**

Two converging lenses serve as an expander for a coaxial Laser beam. The beam (diameter, D_{in} = 1 mm) enters the first L⁺ (f_1 = 30 mm) and emerges from the second L⁺ with D_{out} = 8 mm. Determine f_2 and the separation d between the lenses. Draw a ray diagram.

The ingoing and expanded beams are composed of parallel rays. therefore, $s_{o1} = s_{o2} = \infty$. The "infinitely distant object" has a size of y_{o1} = 1 mm, and is focused by L_1 onto F_{i1}. If the object focus of L_2 coincides with F_{i1}, the beam will be parallel again on the far side. The size of the "infinitely distant image" will be y_{i2} = 8 mm if $f_2 = M_T \cdot f_1$ with $M_T = y_{i2}/y_{o1}$. Therefore, f_2 = 240 mm and $d = f_1 + f_2$ = 270 mm.



(The whole instrument is just an astronomical telescope used backwards.)

4. Prism and its Angle of Minimum Deviation **(4 pts)**

Plot a curve of total deviation angle δ versus entrance angle θ_{i1} for a prism with an apex angle $\alpha = 60°$ and refractive index n =1.52 for θ_{i1} ranging from 30° to 90°. (Use any computer graphing method that you prefer.) From the graph determine the angle of minimum deviation δ_m and the corresponding angle of incidence $\theta_{i1,m}$ for which the minimum deviation occurs.

Solve $\delta = \alpha + \theta_{in} + \theta_{out}$ with $\theta_{out} = \arcsin\left(\sin\alpha \cdot \sqrt{n^2 - \sin^2\theta_{in}} - \cos\alpha \cdot \sin\theta_{in}\right)$ and $\alpha = 60°$, $n = 1.52$ as a function of θ_{in} ($\equiv \theta_{i1}$).